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A RATIONALLY-BASED CORRELATION OF MEAN FRAGMENT SIZE FOR DROP SECONDARY BREAKUP

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Abstract—A correlation was derived for the Sauter mean diameter of fragments produced in the bag and multimode drop breakup regimes for drops having Ohnesorge numbers less than 0.1. Development of the correlation focused on the growth of capillary instabilities on the toroidal rim seen during the final stages of bag breakup. The model linked the time scale for drop breakup and the time scale associated with growth of the unstable waves. The instability scale was approximated from the results of linear stability theory for capillary waves on liquid cylinders. The drop breakup scale was based on correlations available in the literature for drops subjected to a rapid (relative to drop deformation time scales) rise in relative velocity. Though development focused on bag breakup, the resultant expression was also shown to correlate the multimode regime data reasonably well.

Key Words: atomization, drop breakup, drop size

1. INTRODUCTION

Data and/or experimental correlations related to secondary drop breakup are of necessity to those developing models of natural and man-made multiphase systems in which drop fragmentation is a potentially significant phenomenon (e.g. rain, agricultural sprays and combustion involving solid and liquid propellants). To understand the fundamental physics of drop fragmentation better, numerous studies focusing on the breakup of accelerating liquid drops in gaseous environments have been conducted and can be found in the literature. In broad terms, these studies focus on one or more of three general areas: requisite conditions for drop fragmentation, time scales of breakup processes, and properties of resultant fragments. Despite the dependence of interphase mass, momentum, and energy transfer on drop size, the third topic has historically received less attention. There is a paucity of correlations in the literature that would allow the modeler to predict fragment sizes. Two recent examples include the maximum stable diameter model of Pilch & Erdman (1987) and a correlation for Sauter mean diameter (SMD) due to Hsiang & Faeth (1992). However, as will be seen in the review that follows, these correlations either perform best or were developed for more energetic breakup regimes. The work to be discussed presently focused on breakup at lower energy regimes, i.e. nearer the critical conditions necessary for fragmentation to occur at all. An SMD correlation was developed based on the argument that breakdown is governed by the growth of unstable waves on the surface of the drop geometry characteristic of this lower energy regime.

As a brief background, studies by Hsiang & Faeth (1992), Wierzba (1990), Loparev (1975), and others reviewed therein have shown that drop breakup can be categorized into several regimes including bag, multimode, and shear breakup. Transitions between regimes are most often correlated in terms of two parameters: the Weber number $\mathrm{We} = \rho_G u_0^2 d_0/\sigma$, the ratio of dynamic pressure/surface tension forces, and the Ohnesorge number $\mathrm{Oh} = \mu_d/(\rho_d d_0 \sigma)^{1/2}$, the ratio of drop viscous/surface tension forces. Here ρ is density, σ is surface tension, and μ is viscosity, while u_0 and d_0 are the initial drop relative velocity and diameter, respectively. The subscripts G and d denote ambient gas and drop liquid properties, respectively. For fixed Oh, a succession of breakup regimes is exhibited as We increases. (See, for example, figure 1 in Hsiang and Faeth 1992.) For $\mathrm{Oh} < 0.1$, the transition We between these regimes are essentially Oh independent. However, for $\mathrm{Oh} > 0.1$, monotonically increasing values of transition We are observed as Oh increases. In addition to viscous forces, the dynamic character of the drop's flow environment has also been shown to have an impact; a slow rise in relative velocity requires a greater critical We for breakup. (See figure 1 of Loparev 1975.) Most researchers have avoided difficulty in characterizing relative velocity profiles simply by subjecting test drops to step changes in relative velocity.

Fragment sizes and/or size distributions have been presented by Hsiang & Faeth (1992, 1993) and Pilch & Erdman (1987). Pilch & Erdman (1987) compiled size data from the literature spanning $10 < \mathrm{We} < 10^5$ though the represented range of Oh is not explicitly stated. Citing insufficient data on which to characterize size distributions over a broad range of We, the authors chose to focus instead on characteristic or average fragment sizes: maximum diameter, mass mean diameter (MMD), and number mean diameter. A notable exception was the data of Komabayasi *et al.* (1964) for bag breakup of water drops. A plot of normalized number distribution versus normalized fragment size for $\mathrm{We} = 13$ and $5\ \mathrm{mm} < d_0 < 7\ \mathrm{mm}$ was presented from which Pilch & Erdman (1987) concluded that bag breakup yields a bimodal fragment size distribution. However the number and scatter of the data points make this conclusion uncertain. The authors developed an algorithm for estimating the maximum stable diameter of a fragment that can survive a breakup event. It was claimed that the MMD could be estimated empirically once the maximum stable diameter had been calculated based on the observation that the mass median diameter is about one-half the maximum stable diameter. No data, however, was presented to support this claim for $\mathrm{We} < 100$. Drop fragmentation was viewed as a multistage process in which fragments continue to break down as long as local conditions are such that the fragment We exceeds the critical value $\mathrm{We_c}$ for breakup. The end of secondary breakup was posited to result from two processes: the multistage breakup into progressively smaller fragments, and fragment acceleration, which decreases the relative velocity. On the basis of this interpretation, Pilch & Erdman (1987) presented the following estimate for maximum stable diameter:

$$d = \mathrm{We_c}\frac{\sigma}{\rho_G u_0^2}\left(1 - \frac{u_b}{u_0}\right)^{-2}. \qquad [1]$$

Here, u_b is the velocity of the fragment cloud when all breakup processes cease. Equation [1] must be used in conjunction with correlations for $\mathrm{We_c}$, the total time required for breakup, and the velocity of the fragment cloud. Applying this scheme to their compiled data, the authors showed that for $\mathrm{We} < 350$ the ratio of observed/predicted maximum sizes was below unity with agreement worsening as We decreased. Recently, Hsiang & Faeth (1992, 1993) presented a more comprehensive study of fragment sizes and size distribution for $\mathrm{We} \leqslant 375$ and $\mathrm{Oh} < 0.1$. Holography was used to measure fragment properties after breakup for drops (water, glycerol solutions, n-heptane, ethyl alcohol, and mercury) subjected to shock wave initiated disturbances in air at normal temperature and pressure. It was found that fragment size distributions after secondary breakup satisfied the universal root normal distribution function (Simmons 1977) in the bag and multimode regimes with $\mathrm{MMD/SMD} = 1.2$. This finding could also be extended to shear breakup provided that the core drop (the remainder of the original drop after shearing off of fragments has ended) is not considered part of the fragment population. Hsiang & Faeth (1992, 1993) thus showed that the fragment size distribution for all breakup regimes from bag to shear, inclusive, could be estimated by specifying a single parameter (e.g. SMD) alone. Accordingly, the authors developed an SMD correlation by considering physics appropriate to the shear breakup regime. Via an analysis of the boundary layer that develops within a drop under the action of external viscous stresses there resulted:

$$\mathrm{We_{SMD}} = \frac{\rho_G u_0^2 \mathrm{SMD}}{\sigma} = C_s\left(\frac{\rho_d}{\rho_G}\right)^{1/4}\left(\frac{\mu_d}{\rho_d d_0 u_0}\right)^{1/2}\mathrm{We}. \qquad [2]$$

Hsiang & Faeth (1992) applied [2] to their measurements of SMD regardless of breakup regime. The value of the empirical constant C_s, which involves various proportionality factors, was found to be 6.2. The data and correlation are shown together in figure 1. Note that several of the multimode breakup data and all of the shear data points exhibit $\mathrm{We_{SMD}}$ of the resultant fragments

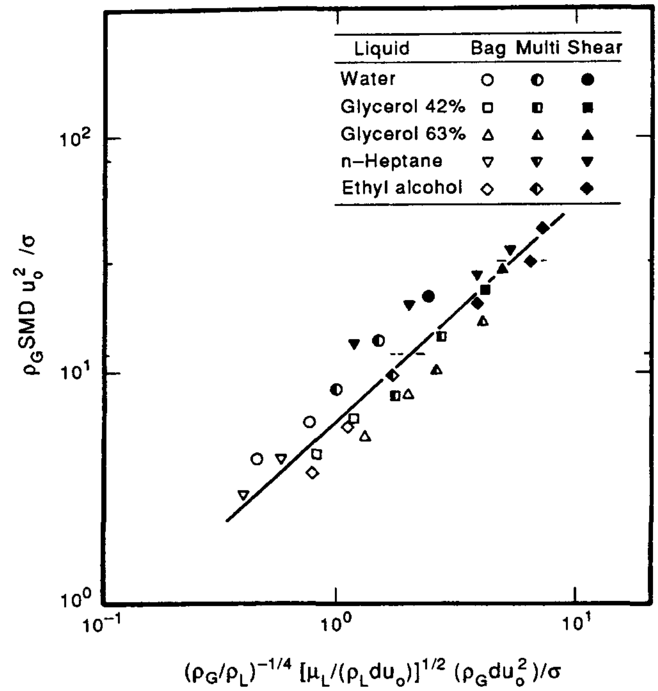

Figure 1. Fragment Sauter mean diameter data; taken from Hsiang & Faeth (1992). Shown also is the predictive equation [2] given by Hsiang & Faeth (1992).

near or above the We_c required for step changes in velocity ($We_c \sim 14$, for $Oh < 0.1$). However, Hsiang & Faeth (1992) noted no further breakdown of larger fragments in the distribution during their experimental observations. Echoing the hypothesis of Pilch & Erdman (1987), this was initially stated to be due to a decrease in relative velocity and fragment size during breakup. In their subsequent work, Hsiang & Faeth (1993) suggested an alternate explanation: that the We_c appropriate to the fragments may be more closely related to that for the onset of breakup for more gradual drop motions, such as studied by Loparev (1975), than to that for a suddenly applied relative velocity, as assumed by Pilch & Erdman (1987). As stated above, Loparev (1975) found that We_c was higher for drops subjected to a slow rise in relative velocity.

The above review of recent fragment size correlations revealed that both were more appropriate for breakup in higher We regimes. In the case of Pilch & Erdman (1987), $We > 350$, while, for Hsiang & Faeth (1992), $We > 90$ (the shear breakup regime). Granted the latter correlation was shown to yield reasonable predictions in the bag and multimode regimes. However it was anticipated that the scatter exhibited for these breakup regimes (see again figure 1) could be reduced by the development of a predictive scheme (or schemes) tailored to their unique physics and/or geometry. To that end the work to be described in subsequent sections aimed to develop a physically-based correlation of the fragment SMD resulting from bag breakup, $12 < We < 30$, for $Oh < 0.1$. Bag breakup is characterized by the initial flattening of the drop into a disc-like shape followed by the development of an indentation on the upstream surface, which leads to a "blowing-up" of the disc into a thin hollow bag having an attached toroidal rim. The thin film of the bag ruptures first yielding a large number of small fragments. Some time later the toroidal rim, which remains after rupture of the film, also breaks up producing fragments several times larger. See Wierzba (1990) or Clift *et al.* (1978) for photographs of this process. The derivation of the correlation focused on the toroidal rim and was founded on the hypothesis that breakdown of the cylindrical filament comprising this structure is due to the presence of unstable waves on the rim surface. Relations for the wave growth rate combined with the time constraints imposed on the process by measured breakup time scales yielded a correlating parameter for the expected fragment size. As characteristic breakup time scales are germane to this study, presentation of available relations will be delayed to a subsequent section.

2. AN IMPROVED SMD CORRELATION FOR THE BAG BREAKUP REGIME

2.1. *Prediction of the expected fragment size via linear stability theory and drop breakup time scales*

Hsiang & Faeth (1992) made the argument for a universal fragment size distribution that can be characterized by a single characteristic diameter (e.g. MMD or SMD), implying that all other characteristic sizes should be interrelated. This hypothesis lends credibility to the development of an SMD correlation based on breakup of the toroidal rim attached to the bag. Empirical evidence suggests that approximately 80% of the original drop mass is comprised by fragments created during this process (Nigmatulin 1991). One could then argue that the mean fragment size resulting from rim dissolution should have a proportionately greater impact on the MMD. Alternately, it is reasonable to expect that this process should yield the maximum-sized fragment in the population. Regardless of which perspective one chooses, the universal distribution championed by Hsiang & Faeth (1992) implies an essentially fixed relationship between SMD and the fragment diameter characteristic of rim breakdown.

Fragmentation of the torus was viewed as being motivated by the growth of capillary instabilities on the surface (see figure 2). From a linear temporal stability analysis of waves on an inviscid liquid cylinder (Chandrasekhar 1961), the time t required for the growth of instabilities necessary to bring about torus breakup is estimated by

$$t \sim \frac{1}{\beta_{\max}} \ln\left(\frac{r}{\delta_0}\right), \qquad [3a]$$

where r is the radius of the torus filament, δ_0 is the initial disturbance amplitude, and

$$\beta_{\max} = 0.34\left(\frac{\sigma}{\rho_d r^3}\right)^{1/2}. \qquad [3b]$$

Here it has been assumed that fragmentation occurs when the amplitude of the least stable wave equals r. An estimate to t can be calculated from two breakup time scales reported in the literature. These are an initiation (or induction) time scale t_i, and a total breakup time scale t_b. The scale t_i is defined as the time required before the deformations that lead ultimately to fragmentation become evident, whereas t_b is the time when the fragmentation event ends and no further breakup occurs. Hence the breakup of the torus must occur in a time $t \sim t_b - t_i$, which constrains r via [3a]

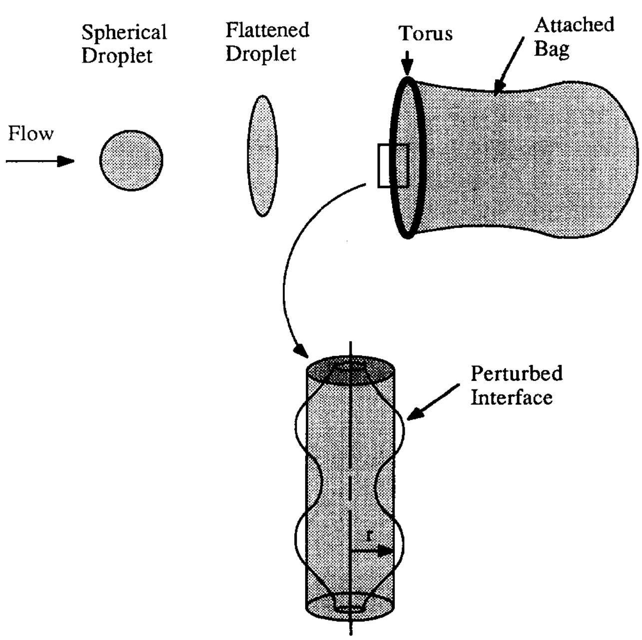

Figure 2. The evolution of the processes leading to drop fragmentation in the bag breakup regime. Shown also is the proposed stability breakup mechanism of the torus attached to the bag.

and [3b]. The least stable wavelength is related to the expected fragment diameter by assuming that each wavelength results in a single fragment (Dombrowski & Johns 1963). A mass balance yields

$$\frac{\pi}{6} d_{\mathrm{f}}^3 = \tfrac{3}{2}\pi r^2 \lambda_{\max}, \qquad [4]$$

where d_{f} is the fragment diameter and $\lambda_{\max}$ is the fastest growing wavelength. Inviscid stability theory predicts (Chandrasekhar 1961) that $\lambda_{\max} = 9r$ leaving $d_{\mathrm{f}}^3 = 81r^3$. Substituting the above relations for t, $\beta_{\max}$, and r (save in the natural log term) into [3a] yields

$$\frac{d_{\mathrm{f}}}{d_0} \sim \left[\frac{3\dfrac{t_{\mathrm{b}} - t_{\mathrm{i}}}{t^*}}{\mathrm{We}^{1/2} \ln\left(\dfrac{r}{\delta_0}\right)}\right]^{2/3} . \qquad [5]$$

for the normalized fragment size. Note that a characteristic scale $t^* = d_0(\rho_{\mathrm{d}}/\rho_{\mathrm{G}})^{1/2}/u_0$ (Ranger & Nicholls, 1969) has been introduced to normalize t_{b} and t_{i}. Equation [5] implies that normalized fragment size increases with increasing breakup time or increasing initial disturbance amplitude. Conversely, increasing We decreases the expected size by a power of $-\frac{1}{3}$.

Equation [5] can be recast in terms of a Weber number We_{f} based on fragment diameter, and converted to an equation by introducing a constant of proportionality C_{t}:

$$\mathrm{We}_{\mathrm{f}} = \frac{\rho_{\mathrm{G}} u_0^2 d_{\mathrm{f}}}{\sigma} = C_{\mathrm{t}}\left[\mathrm{We}\frac{t_{\mathrm{b}} - t_{\mathrm{i}}}{t^*}\right]^{2/3} . \qquad [6]$$

Note that the logarithm of the fractional initial disturbance amplitude has been absorbed into C_{t}. As shown in [3a], this term impacts the time required for breakup. However, it is unclear to the present author how to predict this *a priori*. It is certain that the initial amplitude is a function of the dynamics of the film rupture process and possibly local flow variables, such as ambient turbulence intensity, as well. These influences will not necessarily be identical from one occurrence of drop breakup to the next. Thus, arriving at [6] implicitly assumes that some appropriate mean value of $\ln(r/\delta_0)$ can be specified for the bag regime. Resolution of this issue is a source of possible future work to strengthen the theory.

2.2. Specification of drop breakup time scales

Correlations of t_{i} and t_{b} versus We, for Oh < 0.1, were taken from three sources: the study of Hsiang & Faeth (1992), the review of (primarily) Soviet findings by Nigmatulin (1991), and the data compiled by Pilch & Erdman (1987). The correlations due to Pilch & Erdman (1987) and Hsiang & Faeth (1992) were proposed in their respective papers in reference to drop breakup resulting from sudden exposure of the drop to a high velocity flow field. The flow conditions for the correlations given by Nigmatulin (1991) are not stated specifically but appear also to be for a suddenly applied relative velocity. The applicability of these correlations to other relative velocity histories is uncertain.

For the initiation time, Pilch & Erdman (1987) suggested

$$\frac{t_{\mathrm{i}}}{t^*} = \frac{1.9}{(\mathrm{We} - 12)^{1/4}}, \qquad [7]$$

where t^* is the characteristic time scale introduced above. For total breakup time, separate correlations were specified depending on the We of interest:

$$\frac{t_{\mathrm{b}}}{t^*} = 6(\mathrm{We} - 12)^{-1/4}, \quad \text{for } 12 \leqslant \mathrm{We} \leqslant 18, \qquad [8a]$$

$$\frac{t_{\mathrm{b}}}{t^*} = 2.45(\mathrm{We} - 12)^{1/4}, \quad \text{for } 18 \leqslant \mathrm{We} \leqslant 45, \qquad [8b]$$

$$\frac{t_b}{t^*} = 14.1(\mathrm{We} - 12)^{-1/4}, \quad \text{for } 45 \leqslant \mathrm{We} \leqslant 351. \qquad [8c]$$

It should be noted that the authors provided an Oh correction factor $(1 + 2.2\,\mathrm{Oh}^{1.6})$ for t_i, which is multiplied on the right-hand side of [7] if viscous effects are not negligible. No such correction factor was given for [8a], [8b], and [8c], which are limited to $\mathrm{Oh} < 0.1$. Note also that [8c] corrects a typographical error in the published paper. Nigmatulin (1991) offered

$$\frac{t_i}{t^*} = \frac{2.6(1 + 1.5\,\mathrm{Oh}^{0.74})}{(\ln \mathrm{We})^{1/4}} \qquad [9]$$

for the initiation time and

$$\frac{t_b}{t^*} = \frac{6(1 + 1.2\,\mathrm{Oh}^{0.74})}{(\ln \mathrm{We})^{1/4}} \qquad [10]$$

for total breakup. As with the Pilch & Erdman (1987) t_i correlation, [9] and [10] have multiplicative viscosity correction terms that asymptote to unity with diminishing Oh. Finally, Hsiang & Faeth (1992) suggested

$$\frac{t_i}{t^*} = 1.6 \qquad [11]$$

for the initiation time. The total breakup time was given as

$$\frac{t_b}{t^*} = 5. \qquad [12]$$

The authors provided a correction factor $(1 - \mathrm{Oh}/7)^{-1}$ to be multiplied on the right-hand sides of [11] and [12] if $\mathrm{Oh} > 0.1$. Note that in all cases the effect of increased Oh is to increase the time required for a given breakup event to occur.

2.3. *Evaluation of the proportionality constant C_t*

The constant of proportionality was derived from the fragment size data of Hsiang & Faeth (1992), all of which applied to conditions for which $\mathrm{Oh} < 0.1$. Equation [6] was used with the three sets of time scales presented above. In so doing the fragment diameter d_f assumed the role of the SMD and We_f became $\mathrm{We}_{\mathrm{SMD}}$. The data are listed in table 1. The extraction of C_t from the experimental data was done twice. Using a least-squares technique, C_t was first evaluated by considering the data for the bag breakup regime alone. The resultant coefficients together with their respective relative root mean square (RRMS) errors are given in table 2, for each of the time scale sources. The relative root mean square error was defined as

$$\mathrm{RRMS} = \sqrt{\frac{1}{N}\sum_{1}^{N}\left[\frac{\mathrm{We}_{\mathrm{SMD}} - C_t\,[\mathrm{We}\,t_b - t_i)/t^*)]^{2/3}}{\mathrm{We}_{\mathrm{SMD}}}\right]^2}, \qquad [13]$$

where N is the number of data points considered. The second time, the bag and multimode data were used together in calculating C_t in order to explore the possibility of extending [6] into the multimode regime. The results are presented in table 3.

2.4. *Discussion of results*

The predictions of [6] in conjunction with the coefficients of tables 2 and 3 are shown in figures 3, 4, and 5. Note that the time scales of Pilch & Erdman (1987) resulted in nearly the same value of C_t regardless of which data set was considered. Examining tables 2 and 3, it is seen that the RRMS errors are lowest when only the bag regime data were considered. This is to be expected considering the focus of the derivation. In order to make a valid comparison, the RRMS error was also calculated for [2], the correlation of Hsiang & Faeth (1992). Doing so it was found that the RRMS error equaled 0.315 for the bag regime data and 0.350 for the combined bag and multimode regimes. Considering the focus of [6] on the physics of bag breakup it is understandable that it should provide superior correlation of the data when compared with [2]. However the new correlation also outperforms the shear-based correlation in the multimode regime, though this regime exhibits more complicated structures and often no toroidal rim is observed.

There are a number of possible explanations for the failure of the present correlation to collapse the data to a single line. One obvious possibility would be experimental uncertainty. Consider the ethyl alcohol and the 42% glycerol drops. It is well established that the Weber and Ohnesorge numbers are primary parameters controlling drop fragmentation. Yet despite having nearly equal Ohnesorge and Weber numbers (14.14 for ethyl alcohol and 14.60 for the glycerol), the SMD-based Weber number was 3.67 for ethyl alcohol and 4.45 for the glycerol, a difference of about 20%. One would expect that two drops having nearly equal We and Oh, which suggests dynamic similarity, would behave similarly when they fragment. Perhaps, however, other factors are at work. Recall that in the derivation of the time-based model the term $\ln(r/\delta_0)$ was absorbed into the proportionality constant. As stated previously, it is probable that this term is a function of local flow variables (e.g. ambient turbulence intensity) and the rupture of the film, neither of which were considered in this analysis. Furthermore, errors may be present due to an oversimplified view of the stability breakdown process, which neglected nonlinear and viscous effects. Finally, the assumption that the ratio of SMD for drop breakup to d_f for rim breakdown is a constant may not be strictly correct.

Table 1. Data used in deriving the proportionality coefficient C_t of [6]. Data abstracted from Hsiang & Faeth (1992). P & E denotes time scales of Pilch & Erdman (1987); Nig denotes time scales of Nigmatulin (1991); H & F denotes time scales of Hsiang & Faeth (1992)

Fluid: Oh	We	We_{SMD}	$[We(t_b - t_i)/t^*]^{2/3}$: P & E	$[We(t_b - t_i)/t^*]^{2/3}$: Nig	$[We(t_b - t_i)/t^*]^{2/3}$: H & F
Water:	15.26	4.24	12.94	11.88	13.92
Oh = 0.336×10^{-2}	31.10	5.98	25.80	18.38	22.37
	44.53	8.34	37.01	22.96	28.42
	75.57	13.4	47.42	31.96	40.43
n-Heptane:	10.19	2.89	—	9.35	10.63
Oh = 0.477×10^{-2}	16.99	4.25	12.95	12.72	14.95
Ethyl alcohol:	14.14	3.67	13.19	11.51	13.24
Oh = 0.115×10^{-1}	22.74	5.72	18.09	15.36	18.17
	39.96	9.47	33.29	21.76	26.46
Glycerol, 42%:	14.60	4.45	13.04	11.74	13.52
Oh = 0.119×10^{-1}	23.78	6.37	19.10	15.80	18.72
	39.72	7.89	33.08	21.70	26.35
	73.26	14.46	46.72	31.80	39.63
Glycerol, 63%:	13.15	5.25	13.89	11.34	12.64
Oh = 0.359×10^{-1}	22.75	7.95	18.07	15.83	18.22
	32.81	10.0	27.24	19.84	23.25
	59.09	16.23	42.26	28.62	34.42

Table 2. Coefficient and RRMS error for [6] calculated using the bag breakup regime data of Hsiang & Faeth (1992)

Source of time scale correlations	Coefficient C_t	RRMS error
Pilch & Erdman (1987)	0.318	0.173
Nigmatulin (1991)	0.381	0.154
Hsiang & Faeth (1992)	0.324	0.158

Table 3. Coefficient and RRMS error for [6] calculated using the bag and multimode breakup regime data of Hsiang & Faeth (1992)

Source of time scale correlations	Coefficient C_t	RRMS error
Pilch & Erdman (1987)	0.319	0.194
Nigmatulin (1991)	0.457	0.249
Hsiang & Faeth (1992)	0.374	0.223

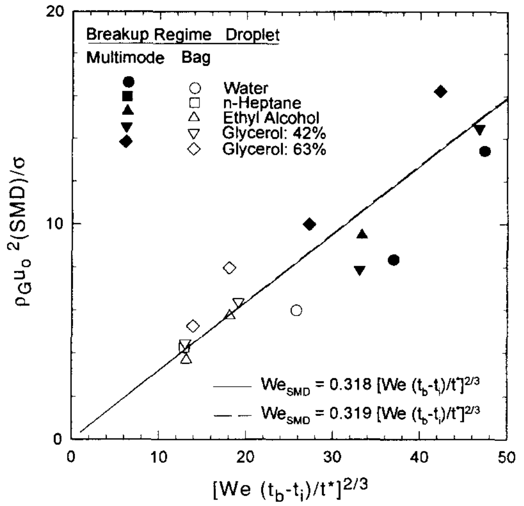

Figure 3. The fragment Sauter mean diameter data of Hsiang & Faeth (1992) plotted vs [6]: —— indicates C_t calculated using the bag regime data only; — — — indicates C_t calculated using both bag and multimode regime data. The correlations of Pilch & Erdman (1987) were used to evaluate the time scales.

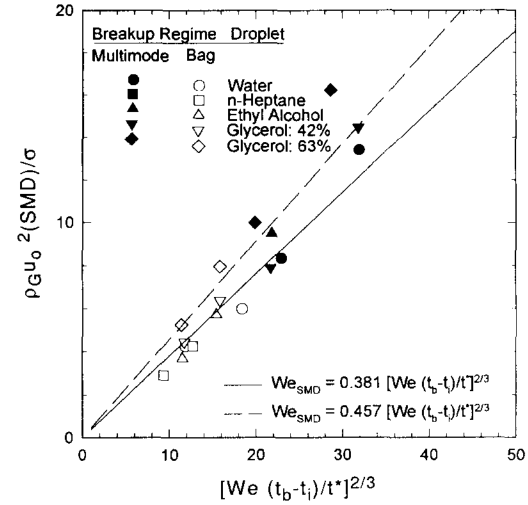

Figure 4. The fragment Sauter mean diameter data of Hsiang & Faeth (1992) plotted versus [6]: —— indicates C_t calculated using the bag regime data only; — — — indicates C_t calculated using both bag and multimode regime data. The correlations of Nigmatulin (1991) were used to evaluate the time scales.

3. CONCLUSIONS

A new correlation was developed to predict fragment SMD for drop breakup in the bag and multimode regimes ($14 < We < 90$), for $Oh < 0.1$. The correlation was based on linear stability theory and correlations of breakup time scales, which were found in the literature. The model contains a constant of proportionality that was derived from the experimental data of Hsiang & Faeth (1992). In comparison with the shear-based correlation of Hsiang & Faeth (1992), the stability/time-based relation offers a noticeable improvement in ability to correlate the bag and multimode regime data. Equation [6] is preferable for predictions in these regimes. The time scales suggested by Pilch & Erdman (1987) and the associated $C_t = 0.32$ are recommended as they yield the lowest RRMS error for the combined bag and multimode regime data set. Thus the recommended relations are:

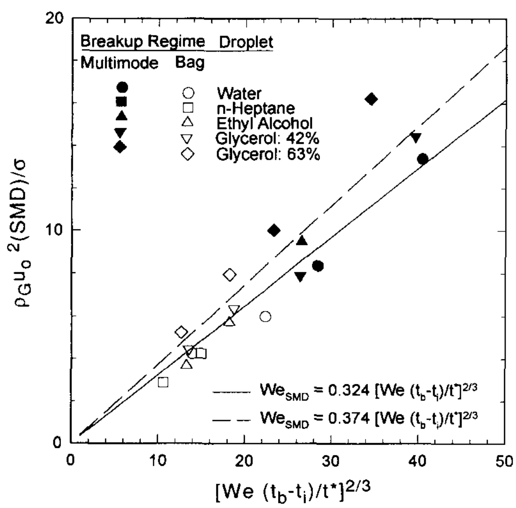

Figure 5. The fragment Sauter mean diameter data of Hsiang & Faeth (1992) plotted versus [6]: —— indicates C_t calculated using the bag regime data only; — — — indicates C_t calculated using both bag and multimode regime data. The correlations of Hsiang & Faeth (1992) were used to evaluate the time scales.

$$\mathrm{We}_{\mathrm{SMD}} = 0.32\mathrm{We}^{2/3}\left[\frac{4.1}{(\mathrm{We}-12)^{1/4}}\right]^{2/3}, \quad \text{for } 12 < \mathrm{We} < 18, \qquad [14a]$$

$$\mathrm{We}_{\mathrm{SMD}} = 0.32\mathrm{We}^{2/3}\left[\frac{2.45(\mathrm{We}-12)^{1/2}-1.9}{(\mathrm{We}-12)^{1/4}}\right]^{2/3}, \quad \text{for } 18 < \mathrm{We} < 45, \qquad [14b]$$

$$\mathrm{We}_{\mathrm{SMD}} = 0.32\ \mathrm{We}^{2/3}\left[\frac{12.2}{(\mathrm{We}-12)^{1/4}}\right]^{2/3}, \quad \text{for } 45 < \mathrm{We} < 100. \qquad [14c]$$

Equations [14] employ auxiliary relations and/or empirical constants which were derived from data pertaining to drop breakup under conditions of a rapid (relative to drop deformation time scales) rise in relative velocity. Thus the accuracy is uncertain for other relative velocity histories. Furthermore, the relations are limited to conditions for which $\mathrm{Oh} < 0.1$. Experimental SMD data at higher Oh is required before the possiblity of extension to more viscous cases can be considered.

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REFERENCES

Chandrasekhar, S. 1961 *Hydrodynamic and Hydromagnetic Stability*. Oxford University Press.

Clift, R., Grace, J. R. & Weber, M. E. 1978 *Bubbles Drops and Particles*. Academic Press.

Dombrowski, N. & Johns, W. R. 1963 The aerodynamic instability and disintegration of viscous liquid sheets. *Chem. Eng. Sci.* **18,** 203–214.

Hsiang, L.-P. & Faeth, G. M. 1992 Near-limit drop deformation and secondary breakup. *Int. J. Multiphase Flow* **18,** 635–652.

Hsiang, L.-P. & Faeth, G. M. 1993 Drop properties after secondary breakup. *Int. J. Multiphase Flow* **19,** 721–735.

Komabayasi, M., Gonda, T. & Isono, K. 1964 Life time of water drops before breaking and size distribution of fragment drops. *J. Met. Soc. Jpn* **42,** 330–340.

Loparev, V. P. 1975 Experimental investigation of the atomization of drops of liquid under conditions of a gradual rise of the external forces. *Iz. Akad. Nauk SSSR Mekh. Zhidk. Gaza* **3,** 174–178.

Nigmatulin, R. I. 1991 *Dynamics of Multiphase Media*. Hemisphere, Washington, DC.

Pilch, M. & Erdman, C. A. 1987 Use of breakup time data and velocity history data to predict the maximum size of stable fragments for acceleration-induced breakup of a liquid drop. *Int. J. Multiphase Flow* **13,** 741–757.

Ranger, A. A. & Nicholls, J. A. 1969 The aerodynamic shattering of liquid drops. *AIAA J.* **7,** 285–290.

Simmons, H. C. 1977 The correlation of drop-size distributions in fuel nozzle sprays. *J. Eng. Power* **99,** 309–319.

Wierzba, A. 1990 Deformation and breakup of liquid-drops in a gas stream at nearly critical Weber numbers. *Exp. Fluids* **9,** 59–64.